use is grossly incorrect, since it disregards an essential additional parameter—the number of different curves fitted.

Use of the variance-ratio test instead of the bias-ratio criterion tends to result in the selection of functions with too many constants. Being right for the wrong reasons, the comment indicates the cubic as most suitable for the example in the paper. The second-best choices are more revealing about the two approaches. The paper selects the first-degree polynomial while the comment prefers the fourth. Even a visual examination should suffice for determining which of them has subset regression curves that might more reasonably be extrapolated to the regions to which they had not been fitted.

For curves that oversmooth data, B is a maximum-likelihood estimator of the ratio of the sum of squares of deviations for a given curve to the sum of squares of deviations for the most suitable, unbiased, even though unknown curve. For the oversmoothed condition, B is measured on a true ratio scale, which admits all arithmetical and statistical operations. In particular, the distance (B-1) is then a measure of the residual, potentially removable bias in terms of irremovable random variation.

The bias-ratio criterion has been in frequent use with various kinds of data, and no difficulty has arisen from the fact that probabilities are not available for the B-s. While some readers might feel more at ease in the probability domain, I know of no valid way of getting there at present.

Comment on "Analysis of Embedded Shock Waves Calculated by Relaxation Methods"

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MURMAN has introduced a new finite-difference operator for calculating embedded shock waves by relaxation methods.[1] It constitutes a significant refinement of the earlier version of the relaxation scheme developed by Murman and Cole.[2] The purpose of this Comment is to substantiate a rationale for the difference in shock strength and shock location generated by using the new and old "shock-point" operators.

Although there might still be some question regarding the consistency condition on the differential equation, the use of the new shock-point operator, Eq. (11) of Ref. 1, is acceptable to the extent that it satisfies the integral equation. The Murman numerical results show that the solution using the new operator—fully conservative relaxation (FCR)—has a stronger shock and one that is considerably farther aft than obtained by using the old elliptic operator—not fully conservative relaxation (NCR). The trend has been consistent throughout all the data presented. Murman has given no reason for such differences or no discussion other than some equivocal remarks on the attribution to the re-expansion singularity. Of course, complaints about the experimental tests or small disturbance equations are irrelevant to the present issue.

In fact, the situation whereby different shock strength and location computed by pure inviscid theory but with different shock relations are encountered is mathematically rigorous and physically realistic. This was experienced at NSRDC when the effect of entropy change across an embedded shock wave was evaluated.[3] However, the numerical scheme used there was the method of integral relations rather than the finite-difference technique. In the case of a finite increase in entropy across the the shock wave, it was found that the shock strength was weakened and the shock location moved forward as shown in Figs. 1 and 2. These figures are taken from Tai[3] (originally Figs. 6 and 7) and include data from Magnus and Yoshihara,[4] Stivers,[5] Steger and Lomax,[6] and Graham et al.[7] The increase in entropy corresponds to the decrease in total pressure. A simple relation between the total pressure and entropy change can be found in Liepmann and Roshko.[8] Mathematically, the inclusion of entropy change provides a rigorous inviscid model to account for the rotational term in the full inviscid flow equations. Physically, the change of entropy creates vorticity behind the shock wave; this has a cumulative effect in the far downstream and consequently feeds back to the shock itself.[3]

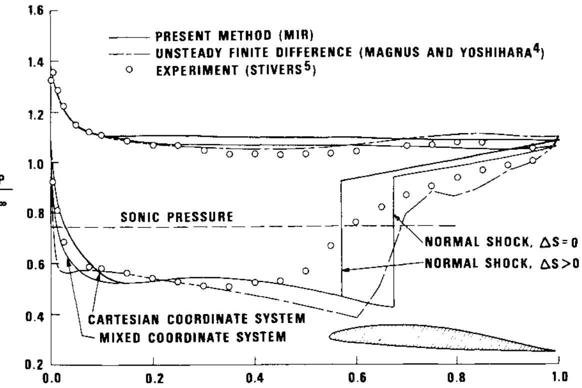

Fig. 1 Pressure distribution on an NACA 64A410 airfoil at $M_\infty = 0.72$ and $\alpha = 4°$.

As a matter of fact, the spurious sinklike terms of Eq. (18b) of Ref. 1 have an effect similar to a decrease in total pressure or an increase in entropy. Since the old elliptic operator is solely responsible for the rise of these sinklike terms, it is apparent that its use gives a weaker and earlier shock wave than one computed by the new shock-point operator. It is interesting to note that the unsteady finite-difference results, which agree well with the solution obtained by using the FCR method in Ref. 1, correspond to the potential flow solution ($\Delta S = 0$) in Fig. 1. On the other hand, those of the relaxation method based on the previous version lie close to the nonisentropic solution (normal shock, $\Delta S > 0$) in Fig. 2. This provides further evidence that the new FCR method has the capability of representing the true potential flow solution.

In any case, the similar effect obtained by using the old elliptic operator and incorporating the entropy change is just a coincidence; their mathematical bases are completely different.

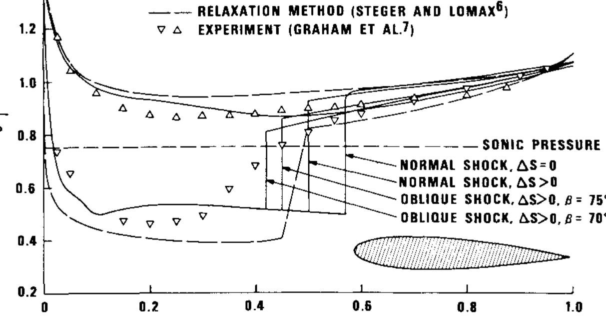

Fig. 2 Pressure distribution on an NACA 0015 airfoil at $M_\infty = 0.729$ and $\alpha = 4°$.

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It is also dangerous to deliberately use the elliptic operator to account for the entropy change of a full inviscid flow; the magnitude of those sinklike terms does not necessarily correspond to that of the total pressure loss determined by the Rankine–Hugoniot relations. The application of the new shock point operator introduced by Murman[1] should certainly be considered as more correct and consistent with transonic inviscid theory, even though its resulting solution deviates more from the experimental data. There are reasons to believe that the large discrepancy between the new solution and the experimental measurement is due to the inherent drawback of the transonic potential flow theory which excludes the effects of entropy change and viscosity.

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Reply by Author to T. C. Tai

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TAI discusses the effect of shock wave strength on shock wave location and offers results showing that a weaker shock wave will be farther forward on an airfoil, everything else being equal. To the author's knowledge, this trend agrees with all experimental and theoretical evidence. It is well known that different sets of governing equations approximating the Euler equations have correspondingly different shock jump relations which provide various approximations to the strength of a Rankine-Hugoniot shock wave. In addition, the shock wave strength will be altered by interaction with the boundary layer. The weakening of the shock wave calculated by the NCR method has no basis as a rational physical or mathematical model of any of these effects. Tai is correct in bringing additional attention to this fact and that the correct solution to the equations is provided by the FCR method.

Tai's concluding statement requires some qualification. The two examples he presents are subject to nonisentropic and strong viscous effects. The shock Mach numbers, M_s, in Figs. 1 and 2 are approximately 1.44 and 1.34, respectively, which exceed the usually quoted upper bound of 1.3 for validity of the isentropic assumption. The data in both figures are for a Reynolds number of $\sim 2\times10^6$ and laminar or transitional type of shock wave boundary-layer interaction is indicated. In many aeronautical applications, it is desired to keep $M_s \lesssim 1.3$ to avoid separation of the boundary layer by the shock wave. Hence, a potential flow model should be useful for the inviscid flow calculations. In addition the Reynolds numbers are usually large enough to produce turbulent boundary layers and the shock wave boundary-layer interaction effects will be significantly less than those in Tai's example. Certainly an accurate model of viscous effects must be included in airfoil calculations to yield dependable predictions. In this regard it should be mentioned that the author[1] and others[2] have found that, in the absence of shock wave boundary-layer separation, the reduction in circulation per se below the inviscid value (using the Kutta condition) has a dominant influence on the entire pressure distribution including the shock wave location.

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Errata

Structure of Shock Waves in Cylindrical Ducts

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[AIAA J 11, 1404–1408 (1973)]

THE open circles in the upper part of Fig. 9 should be solid diamond symbols corresponding to case 3. The revised Fig. 9 is presented below.

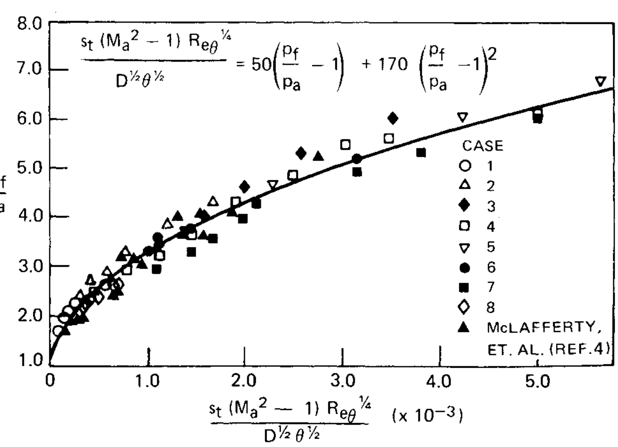

Fig. 9 Correlation of experimental data.

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